

Energy and Heat Transfer Between Two-Dimensional Carrier Systems

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We study how energy and heat are transferred between two two-dimensional carrier systems due to interlayer carrier interaction. We adopt perturbation theory and fluctuation-dissipation relations and derive general expression for the interlayer energy transfer rate, which is applicable to semiconductor and graphene based bilayers. Our detailed calculations for GaAs and Si bilayers suggest that the interlayer energy transfer can exceed the electron-phonon energy transfer below (system dependent) finite crossover temperature. For example, for low mobility Si bilayer with 200 nm layer separation we find a crossover temperature of ~ 1.4 K.

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Solid bodies that are separated by a small vacuum gap can exchange energy and momentum by various mechanisms. When the different bodies represent thermal baths we deal with near-field heat transfer and considerable efforts have been devoted to understand the heat transfer via different channels.¹ One of the most significant exchange channel is built from inter-body photon coupling. Surface excitations involving optical phonons and plasmons can also play an important role and these so-called polariton effects can strongly enhance the heat transfer. Recently, also a near-field heat transfer channel arising directly from lattice vibrations has been proposed.²

Many near-field experiments are in fact performed in a configuration where there is no vacuum gap in between the different solids. For example, extensive investigations of momentum transfer (or drag) due to Coulomb interaction between two two-dimensional (2D) carrier sys-

tems, that reside in the same heterostructure, have been performed.³ In this work, we investigate how energy and heat are transferred between the layers of such bilayer carrier system due to interlayer Coulomb interaction (see Fig. 1). In contrast to the vacuum gap case now the carriers couple to the same phonon bath and this coupling competes with the interlayer energy transfer [see Fig. 1(b)]. We derive general expression of interlayer energy transfer rate, which is applicable to semiconductor and graphene based bilayers, using perturbation theory and fluctuation-dissipation relations. Our formula for interlayer thermal conductance, G_{12} , shows strong similarity with the drag resistance formulas that are derived with-in similar theoretical framework. Approximation formulas and detailed calculations of G_{12} for semiconductor based bilayers are presented. It is shown that G_{12} can dominate over electron-phonon thermal conductance.

The scattering events depicted in Fig. 1(a) are mediated by interlayer interaction which is described by matrix element M_q . The interlayer Hamiltonian H is given by

$$H = \frac{1}{2A} \sum_{\mathbf{q}} M_{\mathbf{q}} \rho_{1\mathbf{q}}^\dagger \rho_{2\mathbf{q}}, \quad (1a)$$

$$\rho_{L\mathbf{q}} = \sum_{\mathbf{k}} \sum_{\sigma, \sigma'} \sum_{s, s'} c_{\mathbf{k}-\mathbf{q}, s' \sigma'}^\dagger F_{\mathbf{k}-\mathbf{q}, s'}^\dagger F_{\mathbf{k}, s} c_{\mathbf{k}, s \sigma}, \quad (1b)$$

where A is the area, $\rho_{L\mathbf{q}}$ is the electron density operator for layer $L = 1, 2$ and $c_{\mathbf{k}, s \sigma}^{(\dagger)}$ is the electron annihilation (creation) operator. Variables \mathbf{k} , s and σ are wavevector, band index and spin index, respectively (here we will assume spin degeneracy). All electron variables depend on the layer index L , but this is typically not written explicitly (e.g. $\mathbf{k} = \mathbf{k}_L$). Factor $F_{\mathbf{k}, s}$ is defined by the wavefunction of the single particle states and product $F_{\mathbf{k}-\mathbf{q}, s'}^\dagger F_{\mathbf{k}, s}$ defines a band form factor. For an ideal 2D electron system we have $F_{\mathbf{k}, s \sigma} = 1$ and summation over band indices s, s' can be ignored. For graphene we have $F_{\mathbf{k}, s} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & s e^{i\theta_{\mathbf{k}}} \end{pmatrix}^\dagger$, where $s = +1$ and $s = -1$ denote conduction and valence bands, respectively, and $\theta_{\mathbf{k}} = \arctan(k_y/k_x)$.

Next H will be considered as a perturbation Hamilto-

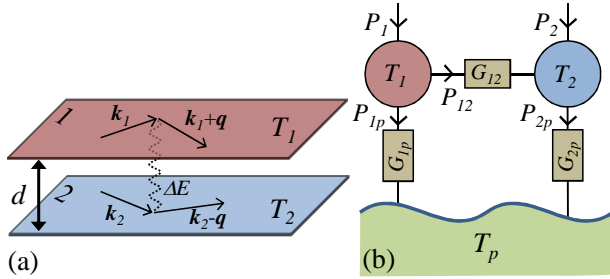


FIG. 1. (color online) (a) Illustration of near-field scattering processes (momentum and energy transfer) between 2D carrier layers 1 and 2 separated by distance d , at local temperatures T_1 and T_2 , respectively. A carrier in layer 1 (2) experiences momentum scattering $\mathbf{k}_1 \rightarrow \mathbf{k}_1 + \mathbf{q}$ ($\mathbf{k}_2 \rightarrow \mathbf{k}_2 - \mathbf{q}$) due to interlayer interaction. During the process energy ΔE is transferred between the layers. (b) The equivalent thermal circuit. P_L is the input heating/cooling power ($L = 1, 2$) and G_{12} is the interlayer thermal conductance. Due to the interlayer scattering processes $G_{12} \neq 0$ and power (or heat) P_{12} flows between the layers. Layers couple to phonon bath, which is at temperature T_p , via electron-phonon thermal conductance G_{Lp} and power P_{Lp} flows to the bath.

nian that will cause transitions from initial state $|i1, i2\rangle = |i1\rangle|i2\rangle$ with energy $E_i = E_{1i} + E_{2i}$ to final state $|f1, f2\rangle = |f1\rangle|f2\rangle$ with energy $E_f = E_{1f} + E_{2f}$. Here $|iL\rangle$ ($|fL\rangle$) is the initial (final) state of layer L . The transition rate Γ_{fi} from initial state i to final state f is given by the golden rule formula

$$\Gamma_{fi} = \frac{2\pi}{\hbar} |\langle f2, f1 | H | i1, i2 \rangle|^2 \delta(E_i - E_f). \quad (2)$$

By multiplying Γ_{fi} by the energy change $\Delta E_1 = E_{1i} - E_{1f}$ and performing an ensemble average over the initial electronic states, and summing over final electronic states we obtain energy transfer rate (heat transfer)

$$P_{12} = (2\pi/\hbar)(2A)^{-2} \sum_{\mathbf{q}} \sum_{i1, f1} \sum_{i2, f2} \Delta E_1 \hat{w}_{1i} \hat{w}_{2i} |M_{\mathbf{q}}|^2 \times \left| \langle f1 | \rho_{1\mathbf{q}}^\dagger | i1 \rangle \right|^2 \left| \langle f2 | \rho_{2\mathbf{q}} | i2 \rangle \right|^2 \times \delta(E_{1i} + E_{2i} - E_{1f} - E_{2f}), \quad (3)$$

where \hat{w}_{Li} is the weighting factor of carrier layer L in state i . We assume that each layer L can be described by a local temperature T_L and, therefore, $\hat{w}_{Lf} = \hat{w}_{Li} \exp[(E_i - E_f)/k_B T_L]$. By using identity $\delta(E_A + E_B) = \hbar \int_{-\infty}^{+\infty} d\omega \delta(E_A - \hbar\omega) \delta(E_B + \hbar\omega)$ and definition of correlator

$$C_L(\mathbf{q}, \omega) = 2\pi\hbar \sum_{n, m} \hat{w}_{Ln} |\langle nL | \rho_{L\mathbf{q}} | mL \rangle|^2 \times \delta(E_{Ln} - E_{Lm} + \hbar\omega) \quad (4)$$

we find

$$P_{12} = \frac{1}{2\pi\hbar^2} \left(\frac{1}{2A} \right)^2 \int_{-\infty}^{+\infty} d\omega \sum_{\mathbf{q}} \hbar\omega |M_{\mathbf{q}}|^2 \times C_1(\mathbf{q}, -\omega) e^{\hbar\omega/k_B T_2} C_2(\mathbf{q}, -\omega). \quad (5)$$

Adopting the fluctuation dissipation relation⁴ $(1 - e^{-\hbar\omega/k_B T_L}) C_L(\mathbf{q}, \omega) = -2\hbar A \text{Im}\{\chi_L(\mathbf{q}, \omega)\}$ and the property $C_L(\mathbf{q}, -\omega) = e^{-\hbar\omega/k_B T_L} C_L(\mathbf{q}, \omega)$ we find the general expression for the interlayer heat transfer

$$P_{12} = \int_0^{+\infty} \frac{d\omega}{2\pi} \sum_{\mathbf{q}} \hbar\omega |M_{\mathbf{q}}|^2 \text{Im}\{\chi_1(\mathbf{q}, \omega)\} \times \text{Im}\{\chi_2(\mathbf{q}, \omega)\} [n_1(\hbar\omega) - n_2(\hbar\omega)], \quad (6)$$

where $n_L(\hbar\omega) = (\exp(\hbar\omega/k_B T_L) - 1)^{-1}$ and $\chi_L(\mathbf{q}, \omega)$ is the susceptibility, which can depend on T_L . At the limit $T_1, T_2 \rightarrow T$ it is useful to define the interlayer thermal conductance $G_{12}(T) = P_{12}/(T_1 - T_2)$. From Eq. (6) we find

$$G_{12}(T) = \frac{\hbar}{4k_B T^2} \int_0^{+\infty} \frac{d\omega}{2\pi} \sum_{\mathbf{q}} (\hbar\omega)^2 |M_{\mathbf{q}}|^2 \times \frac{\text{Im}\{\chi_1(\mathbf{q}, \omega)\} \text{Im}\{\chi_2(\mathbf{q}, \omega)\}}{\sinh^2(\hbar\omega/2k_B T)}, \quad (7)$$

which has a striking similarity with bilayer drag resistance formula.⁵ Equation (7) has only single temperature

and, therefore, it is more convenient to adopt in the case studies below.

We assume that the interlayer interaction is mediated by screened Coulomb interaction, when the matrix element is given by $M_{\mathbf{q}} = \epsilon_{12}^{-1}(\mathbf{q}, \omega) U(\mathbf{q}) F_{12}(d)$, where $U(\mathbf{q}) = e^2/2\epsilon_b q$ is the 2D Fourier transform of Coulomb potential (ϵ_b is the background dielectric constant) and $F_{12}(d)$ is the spatial form factor, which depends on the spatial extent of the electron wave functions and layer distance d . For the sake of simplicity here we assume vanishing extent and as a result we have $F_{12}(d) = \exp(-qd)$. The inter-layer dielectric function $\epsilon_{12}(\mathbf{q}, \omega)$ is given by $\epsilon_{12}(\mathbf{q}, \omega) = [1 - U(\mathbf{q})\chi_1(\mathbf{q}, \omega)][1 - U(\mathbf{q})\chi_2(\mathbf{q}, \omega)] - F_{12}^2 U(\mathbf{q})^2 \chi_1(\mathbf{q}, \omega)\chi_2(\mathbf{q}, \omega)$.⁵

In the ballistic limit the carrier mean free path l_e exceeds the layer distance ($l_e \gg d$) and we use the susceptibility of an ideal 2D electron gas⁶:

$$\chi_L(\mathbf{q}, \omega) = \nu(2z)^{-1} \{2z - \Omega_-(z, u) - \Omega_+(z, u) + F_-(z, u) - F_+(z, u)\}. \quad (8)$$

Here $\Omega_{\pm}(z, u) = C_{\pm} \sqrt{(z \pm u)^2 - 1}$, $F_{\pm}(z, u) = iD_{\pm} \sqrt{1 - (z \pm u)^2}$, $z = q/2k_F$, $u = \omega/qv_F$, $C_{\pm} = (z \pm u)/|z \pm u|$ and $D_{\pm} = 0$ for $|z \pm u| > 1$, and $C_{\pm} = 0$ and $D_{\pm} = 1$ for $|z \pm u| < 1$. Here v_F (k_F) is the Fermi velocity (wave vector) and $\nu = \nu(\epsilon_F)$ is the density of states at Fermi level $\epsilon_F \gg k_B T$. For two similar layers we find asymptotic low-temperature result

$$G_{12}(T) \simeq \frac{f(\kappa d)}{d^{2-\alpha}} \frac{k_B}{(2\pi)^2} \frac{k_B}{v_F^{\alpha}} \left(\frac{k_B T}{\hbar} \right)^{1+\alpha}, \quad (9)$$

where $\alpha = 1.86$, $\kappa = \frac{\nu e^2}{2\epsilon_b}$ is the screening wave vector and $f(a) \simeq 13.66a^{-2}$. The above Equation is valid when $k_F d k_B T < 2E_F$. Parameter κd characterizes the screening of the interlayer interaction: large (small) κd means strong (weak) screening.

In the diffusive limit ($\omega\tau, l_e/d \ll 1$) the susceptibility can be approximated as

$$\chi_L(\mathbf{q}, \omega) \simeq -\nu \frac{iDq^2}{\omega + iDq^2}, \quad (10)$$

where $D = v_F^2 \tau / 2$ is the diffusion coefficient and $\tau = l_e/v_F$ is the momentum relaxation time. By substituting Eq. (10) into Eq. (7) we find analytical approximation

$$G_{12}(T) \simeq \frac{3A_3 \epsilon_b}{16\pi} \frac{1}{d} \frac{1}{2\sigma} \hbar \left(\frac{k_B}{\hbar} \right)^3 T^2. \quad (11)$$

Here $\sigma = e^2 \nu D$ is the DC conductivity of single layer and $A_n = \Gamma(n) \zeta(n) = \int dx x^{n-1} / [\exp(x) - 1]$. Equation (11) is valid when $(l_e/v_F) k_B T / \hbar < (l_e/d)^2$. Note that Eq. (11) applies also for low mobility graphene bilayers.

As depicted in the thermal circuit of Fig. 1(b) G_{12} competes with the electron-phonon thermal conductance G_{Lp} , which at the limit $T_L, T_p \rightarrow T$ is given by $G_{Lp}(T) =$

$P_{Lp}/(T_L - T_p)$. In 2D electron systems at low temperatures the electron-phonon energy transfer is dominated by screened deformation potential (G_{Lp}^{DP}) and piezoelectric (G_{Lp}^{PE}) interaction with total thermal conductance $G_{Lp}(T) = G_{Lp}^{DP}(T) + G_{Lp}^{PE}(T)$. For the deformation potential contribution we have^{7,8}

$$G_{Lp}^{DP}(T) = \sum_{\lambda} \frac{F_{6-n} 2^n}{l_e^n \kappa^2} \langle f_n(\theta) \Xi^2 \rangle v_{\lambda}^{n-6} T^{6-n}, \quad (12)$$

where $n = 0(1)$ represents ballistic (diffusive) limit of electron-phonon coupling, for which we have $q_{\lambda T} l_e > 1 (< 1)$. Here $q_{\lambda T} = k_B T / \hbar v_{\lambda}$ is the thermal phonon wave vector, factor $F_i = \frac{\nu A_i}{2\pi^2 \rho v_F} \frac{(i+1)k_B^{i+1}}{\hbar^i}$, $v_{L(T)}$ is the longitudinal (transversal) phonon velocity and ρ is the mass density of the crystal. The brackets $\langle \dots \rangle$ stand for solid angle average and θ is the angle with respect to the z-axis, which is perpendicular to the layer plane. $\langle f_n(\theta) \Xi^2 \rangle$ is an effective deformation potential coupling and $f_0(\theta) = \sin \theta$ and $f_1(\theta) = \frac{\sin^2 \theta}{\alpha + \sin^2 \theta}$. Parameter $\alpha = (\kappa l_e v_F / v_{\lambda})^{-2}$ and as a result $f_1(\theta) \approx 1$. The piezoelectric coupling gives rise to contribution⁷⁻⁹

$$G_{Lp}^{PE}(T) = \sum_{\lambda} \frac{F_{4-n} 2^n}{l_e^n \kappa^2} \langle f_n(\theta) K^2 \rangle v_{\lambda}^{n-4} T^{4-n}, \quad (13)$$

where $\langle f_n(\theta) K^2 \rangle$ is the effective piezo coupling.

Figure 2 shows $G_{12}(T)$ obtained numerically from Eqs. (7) and (8) in the case of symmetric high mobility (ballistic) GaAs-AlGaAs bilayer¹⁰ (here referred as GaAs bilayer) with single layer electron density $n = 10^{15} \text{ m}^{-2}$ and $d = 20 \text{ nm}$. Asymptotic limit formula of

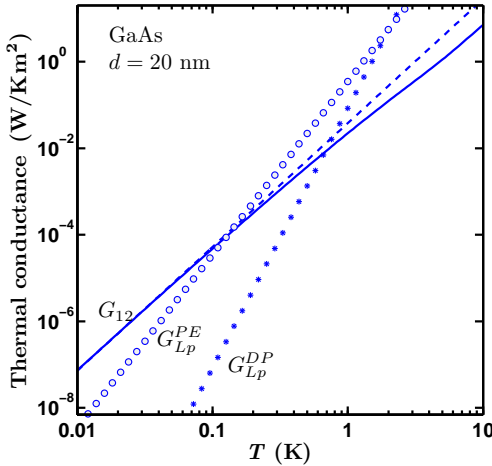


FIG. 2. (color online) The interlayer thermal conductance (G_{12}) and deformation potential (G_{Lp}^{DP}) and piezoelectric (G_{Lp}^{PE}) electron-phonon thermal conductance at the ballistic limit for GaAs bilayer system with electron density $n = 10^{15} \text{ m}^{-2}$. Solid curve is the result from numerical integration using the ballistic response function. Dashed curve obtained using Eq. (9).

Eq. (9) is also plotted. In the phonon contribution we have $\sum_{\lambda} \langle \sin \theta \Xi^2 \rangle v_{\lambda}^{-6} = \frac{1}{4} \pi \Xi_d^2 v_L^{-6}$, where $\Xi_d = 10 \text{ eV}$ is the dilatational deformation potential constant, and $\sum_{\lambda} \langle \sin \theta K^2 \rangle v_{\lambda}^{-4} = \left(\frac{e}{\epsilon_b} \right)^2 e_{14}^2 \pi \left(\frac{89}{1024} v_L^{-4} + \frac{107}{1024} v_T^{-4} \right)$, where $e_{14} = -0.16 \text{ C/N}$ is the only non-zero element of the piezotensor. Other parameters can be found from Ref.¹¹. Equations (12) and (13) are plotted as symbols in Fig. 2 in the ballistic limit. Below few Kelvin piezoelectric coupling fully dominates and as a result the temperature regime where $G_{Lp} < G_{12}$ is pushed towards relatively low, but experimentally achievable, temperatures. The crossover occurs at $T \sim 140 \text{ mK}$.

For silicon based bilayer¹² we consider both ballistic and diffusive limits at electron density $n = 5 \times 10^{15} \text{ m}^{-2}$. Parameters for Si can be found from Ref.¹¹. The curves in Fig. 3 are calculated for symmetric high (low) mobility Si bilayer system with mobility $\mu = 2.5 (0.2) \text{ m}^2/\text{Vs}$, mean-free path $l_e = 200 (16) \text{ nm}$ and layer distance $d = 20 (200) \text{ nm}$. For the high mobility device we have used the ballistic limit response [Eq. (8)] and for the low mobility one the diffusive response [Eq. (10)]. Even though the layer distance is one magnitude larger in the diffusive case, G_{12} is still significantly larger than in the ballistic case. Thus, disorder can strongly enhance the interlayer energy and heat transfer. Silicon is not piezoelectric so for G_{Lp} we need to consider only G_{Lp}^{DP} . Due finite electron mean free path (even for the high mobility device) we will include ballistic and diffusive limits of G_{Lp}^{DP} . For simplicity we plot G_{Lp}^{DP} so that it changes abruptly from diffusive to ballistic formula [note that Eq. (12) is valid close to $q_{\lambda T} l_e = 1$].

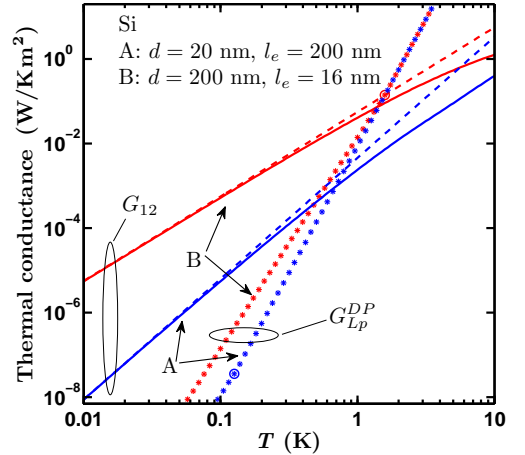


FIG. 3. (color online) The interlayer thermal conductance (G_{12}) and deformation potential (G_{Lp}^{DP}) electron-phonon thermal conductance for two Si bilayer devices with $n = 5 \times 10^{15} \text{ m}^{-2}$. For device A (B) solid curves is a result from numerical integration using the ballistic (diffusive) susceptibility and dashed curves are obtained from the approximation formula of Eq. (9) (Eq. (11)). The circle marks the crossover where electron-phonon coupling changes from ballistic to diffusive.

For Si 2D electron gas we have $\sum_{\lambda} \langle \sin \theta \Xi^2 \rangle v_{\lambda}^{-6} = \frac{1}{32} \pi (4\Xi_d \Xi_u + 8\Xi_d^2 + \Xi_u^2) v_L^{-6} + \frac{1}{32} \pi \Xi_u^2 v_T^{-6}$ and $\sum_{\lambda} \langle \Xi^2 \rangle v_{\lambda}^{-6} = (\frac{2}{3} \Xi_d \Xi_u + \Xi_d^2 + \frac{1}{5} \Xi_u^2) v_L^{-6} + \frac{2}{15} \Xi_u^2 v_T^{-6}$, where Ξ_u is the uniaxial deformation potential constant. We use the typical values $\Xi_{d(u)} = -11.7(9.0)$ eV. For the high and low mobility Si systems the crossover temperature where $G_{12} = G_{Lp}$ is 660 mK and 1.4 K, respectively.

Another widely explored bilayer carrier system, which can be realized using compound semiconductors¹³ or Si¹⁴, is the electron-hole bilayer. The complexity of the valence band makes this system more difficult to analyze theoretically. We will not present G_{12} for such system here explicitly, but it should behave in similar fashion as its electron-electron counter part. However, one thing that may differ drastically from electron-electron bilayer system is the carrier-phonon coupling. Due to asymmetry in the deformation potential coupling between the different layers the carrier-phonon coupling can be unscreened and as a result G_{Lp} can be strongly enhanced at low temperatures.⁸ The enhancement factor depends on the details, but in many cases it is of the order of $(\kappa/q_{\lambda T})^2$, which suggests that for electron-hole bilayers G_{Lp} can dominate over G_{12} even down to very low temperatures. Note that also in symmetric electron bilayers G_{Lp} can be affected by the presence of another carrier system in non-trivial way, but significant enhancement is not expected⁸.

The interlayer heat transfer can be investigated experimentally by varying the input powers P_L while measuring the electron temperatures T_L [see Fig. 1(b)]. Uniform input power follows, for example, from Ohmic heating. The electron-phonon contributions G_{Lp} can be investigated independently from G_{12} by depleting the other carrier system and/or at balanced input power that gives $T_1 = T_2$. It is not necessarily trivial to measure electron temperature of the individual layers. For exam-

ple, quantum corrections of resistivity and Shubnikov-de Haas oscillations, that have been used as electron thermometer, can be affected by the other layer in non-trivial way. More local temperature probes based, e.g., on quantum point contacts and quantum dots¹⁵ have also been demonstrated. Especially in the case of Si, one possibility is to control P_L and measure temperature in doped contact regions, when G_{12} can be investigated by adjusting layer densities by gate voltage. Electron-phonon coupling in doped semiconductors can be relative weak so that G_{12} dominates. However, it is important to note (as already pointed out in Ref.⁸) that similar near-field thermal coupling to G_{12} can exist between bilayer carriers and external gate electrodes. The sign of P_L can be also reversed which is equivalent to cooling. This can be achieved by quantum dots¹⁵ or semiconductor-superconductor contacts¹⁶. In general, G_{12} could be used as a controlled thermalization path (thermal switch) at low temperatures.

In summary, by using first order perturbation theory and fluctuation-dissipation relations we derived a general expression of inter-layer energy transfer rate [Eq. (6)] and thermal conductance [Eq. (7)]. Our formulation can be applied, for example, to semiconductor and graphene based bilayers. We presented analytical approximation formulas and detailed calculations for GaAs and Si based bilayers. Interlayer energy transfer was compared to electron-phonon coupling and our results showed that the inter-layer energy transfer exceeds the electron-phonon energy transfer below crossover temperature of 140 mK (660 mK) for ballistic GaAs (Si) bilayer with $d = 20$ nm layer distance. Interlayer energy transfer is enhanced in the diffusive limit and for low mobility Si bilayer with $d = 200$ nm the crossover occurs at ~ 1.4 K.

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